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High-dimensional sources for the four-dimensional gravity

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We argue that a certain distribution of matter in higher dimensions can provide the correct behaviour of gravity in four dimensions. Some explicit examples illustrating the idea are considered.

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimensions beyond those four observed so far. The absence of any signature of extra dimensions in current experiments is usually explained by the compactness of extra dimensions. Remarkably, the size of such dimensions can be macroscopically large ($\sim$ 1mm), if the only known particle filling the extra dimensions is a graviton, while the other particles are localized on the four-dimensional submanifold (3-brane) embedded in higher-dimensional space-time [1]. Alternatively, one can consider high-dimensional space-time with non-factorizable geometry where the four-dimensional Newton law is correctly reproduced even in the case of non-compact extra dimensions [2,3] (for an earlier related works, see [4] and for some recent extensions and modifications, see [5–10]). The source for such a gravity is the 3-brane embedded into the five-dimensional anti-de Sitter space-time with the tension finely tuned to the value of the five-dimensional negative cosmological constant [2,3].

In this paper we analyze more general high-dimensional sources which indeed lead to the correct behaviour of gravity in four dimensions. Let us start with five-dimensional Einstein's equations:

$$G_{AB} \equiv R^{(5)}_{AB} - \frac{1}{2}g_{AB}R^{(5)} = k_5^2 T_{AB}, \tag{1}$$

where $k_5^2 = 8\pi G_N^{(5)} = 8\pi/M_5^3$ and $A, B = 0, ..., 4$. Taking the anzatz [1]

$$ds^2 = a^2(x^4)\eta_{\alpha\beta}dx^\alpha dx^\beta + (dx^4)^2, \tag{2}$$

for the line-element of the five-dimensional space-time, which respects the four-dimensional Poincaré invariance, Eqs.(1) decompose as:

$$a^{-2}G^\alpha_\beta + 3\left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2\right]\delta^\alpha_\beta = k_5^2 T^\alpha_\beta, \tag{3}$$

$$-a^{-2}R + 6\left(\frac{a'}{a}\right)^2 = k_5^2 T^5_5, \tag{4}$$

where G^α_β and R are the four-dimensional Einstein tensor and Ricci scalar, respectively, built up from the four-dimensional metric $\eta_{\alpha\beta}$ $(\alpha, \beta = 0, ..., 3)$ and the prime denotes differentiation with respect to the extra coordinate x^4. From (3) and (4) it is obvious that in order to have correct Einstein's equations in four dimensions (i.e. the correct four-dimensional gravity), we should have

$$G^\alpha_\beta = k_4^2 \tau^\alpha_\beta, \tag{5}$$

where ordinary four-dimensional Newton's constant $k_4^2 = 8\pi G_N = 8\pi/M_{Pl}^2$ is related to the five-dimensional one according to

$$k_5^2 = k_4^2 \int a^2(x^4)dx^4. \tag{6}$$

Therefore, it is necessary to satisfy the following differential equations:

$$3\left[\frac{a''}{a} + \left(\frac{a'}{a}\right)^2\right]\delta^\alpha_\beta = k_5^2 \tilde{T}^\alpha_\beta, \tag{7}$$

$$6\left(\frac{a'}{a}\right)^2 = k_5^2 \tilde{T}^5_5. \tag{8}$$

Here we identify $\tilde{T}^\alpha_\beta = T^\alpha_\beta - k_4^2 k_5^{-2} a^{-2}\tau^\alpha_\beta$ and $\tilde{T}^5_5 = T^5_5 - k_4^2 k_5^{-2} a^{-2}\tau^\alpha_\alpha$ with the energy-momentum stress tensor of the matter source in extra space, while τ^α_β is the energy-momentum stress tensor of the four-dimensional matter. Usually, one chooses a definite stress tensor that describes a certain distribution of matter in extra dimensions and then looks for the solutions of the system of Eqs. (7) and (8). Obviously, the non-trivial solution of Eqs. (7) and (8) requires a certain relation among the components of the chosen energy momentum

[1] Here we are interested in the static solution. For non-static (cosmological) solutions see the recent review [11] and references therein.

tensor $\tilde{T}^A_B$. For example, the original solution of Refs. [2,3] [2],

$$a^2 = \exp(\pm 2\kappa |x^4|), \tag{9}$$

arises for the energy-momentum stress tensor

$$\tilde{T}^\alpha_\beta = -\Big(\Lambda + \sigma\delta(x^4)\Big)\delta^\alpha_\beta, \tag{10}$$

$$\tilde{T}^5_5 = -\Lambda, \tag{11}$$

if the five-dimensional cosmological constant Λ ($\Lambda < 0$) and the 3-brane [3] tension σ are related as:

$$\sigma = \frac{\kappa}{k_5^2}\ , \quad \kappa = k_5\sqrt{-\frac{\Lambda}{6}}. \tag{12}$$

On the other hand, one can simply consider the Eqs. (7) and (8) just as equations for the source energy-momentum tensor $\tilde{T}^A_B$. Indeed, parameterizing $\tilde{T}^A_B$ as

$$\tilde{T}^\alpha_\beta = \frac{3}{k_5^2}F^2(x^4)\delta^\alpha_\beta, \tag{13}$$

$$\tilde{T}^5_5 = \frac{6}{k_5^2}Q^2(x^4), \tag{14}$$

from (8) one can easily obtain the solution

$$a = \exp\Big\{\int Q(x^4)dx^4\Big\} \tag{15}$$

and from (7), the equation related to the source functions F and Q:

$$Q' + 2Q^2 = F^2. \tag{16}$$

The solution (15) together with the relation (16) is a simple generalization of (9) with (12). Thus, we see, that it is always possible to get a correct behaviour of gravity in four dimensions by choosing the source functions (13) and (14)

[2] The solution with negative exponent [3] corresponds to the case when the gravity is localized on the 3-brane at $x^4 = 0$. Such a 3-brane gravitationally repulses the matter, thus an extra mechanism for the localization of matter on the 3-brane is necessary to be imposed (for recent ideas of localizing matter within the field-theoretic approach, see [12]). Alternatively, for the solution with a positive exponent [2] the 3-brane is gravitationally attractive, while gravity itself is not localized on the 3-brane at $x^4 = 0$ (formally it is localized at infinity).

[3] The 3-brane is considered infinitely thin with a tension picked at $x^4 = 0$. For the discussion of thick branes, see e.g. [13].

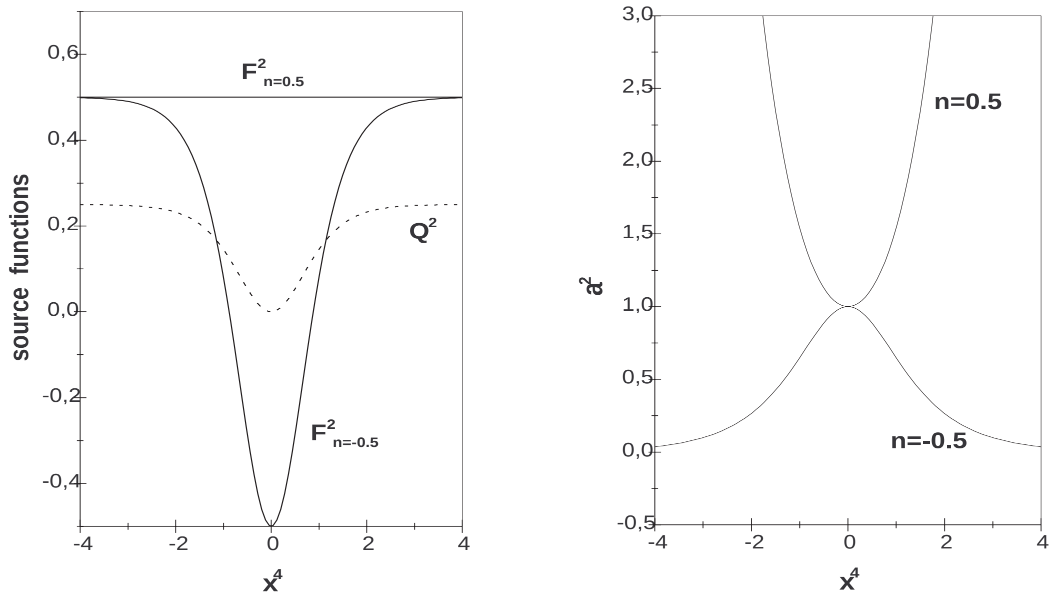

Fig. 1. Source functions (left) and conformal factors (right) for the two particular cases $n = \pm 0.5$ of solutions A.

satisfying (16). However, in order to have a fully consistent picture of all four-dimensional interactions one should consider non-gravitational interactions as well in the high-dimensional background determined by the source functions F and Q. Indeed, each choice of the source functions F and Q requires further investigation in order to clarify in what extent the physics could behave as being effectively four-dimensional, not contradicting the current experiments. The usual way to overcome this problem in the case of non-compact extra dimensions, is to assume that the ordinary matter is localized in four dimensions and thus does not fill the extra dimensions at all. Here we assume that this is the case, not specifying the actual mechanism responsible for such a localization. Now let us consider some explicit solutions which essentially differ from those already considered in the literature.

Solution A. This solution corresponds to the choice of the source functions as:

$$F^2 = \frac{n}{\cosh^2(x^4)}\left[1 + 2n\sinh^2(x^4)\right],$$
$$Q^2 = n^2\tanh^2(x^4), \tag{17}$$

where n is a constant parameter. The solution (up to the integration constant which we set to be equal to 1) for the conformal factor a^2 in (2) is

$$a^2 = \cosh^{2n}(x^4). \tag{18}$$

For $n < 0$ the solution (18) is analogous to the solution of Refs. [3] with graviton zero mode localized in four dimensions at $x^4 = 0$. For $n > 0$ we

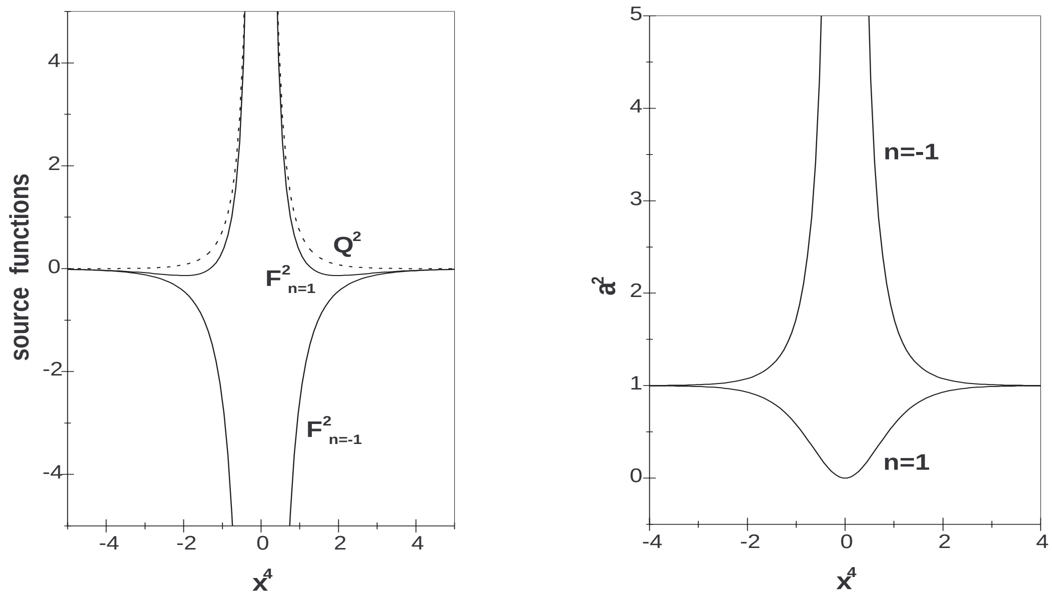

Fig. 2. The same as in Fig. 1 for the two particular cases $n = \pm 1$ of solutions B.

obtain a solution which grows as we go away from the point $x^4 = 0$, thus providing an attractive gravitational potential for trapping the matter in four dimensions around $x^4 = 0$ [2]. We have plotted the functions F^2, Q^2 and the conformal factor a^2 in Fig. 1 for $n = \pm 1/2$. Note, that for $n = +1/2$, $F^2 = 1/2$ is constant, while Q^2 drops to its minimum at $x^4 = 0$. Such a distribution of matter is somewhat opposite to the original scenarios of Refs. [2,3] (see Eqs. (10) and (11)).

Solution B. Now let us consider the following source functions:

$$F^2 = n^2 \mathrm{csch}^2(x^4)\Big[2n - \cosh(x^4)\Big],$$
$$Q^2 = n^2 \mathrm{csch}^2(x^4), \tag{19}$$

where n is an arbitrary constant parameter again. The conformal factor in this case is:

$$a^2 = \tanh^{2n}(x^4). \tag{20}$$

The source functions (19) are in fact singular at $x^4 = 0$. This leads to a singular behaviour of the induced four-dimensional metric ($a^2\eta_{\alpha\beta}$) at $x^4 = 0$ as well. For $|n| > 1$, however, the conformal factor (20) asymptotically approaches to one, while the source functions go to zero (see Fig. 2). Thus far from the singular point $x^4 = 0$, the space is essentially Minkowskian. One can imagine that the ordinary four-dimensional matter is localized far from the singularity, where the correct four-dimensional gravity could be reproduced.

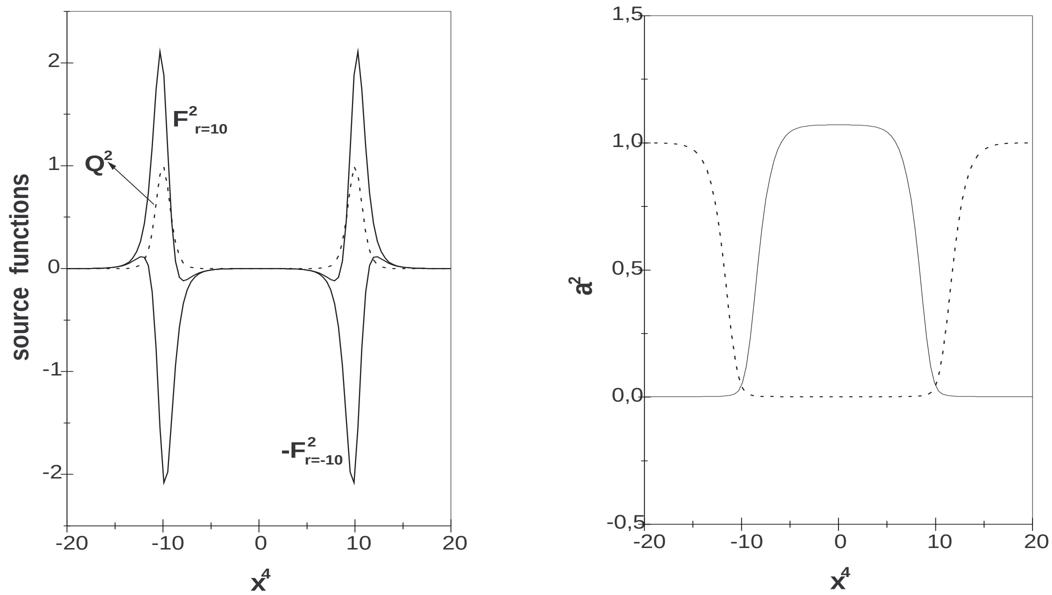

Fig. 3. The same as in Fig. 1 for the two particular cases $r = \pm 10$ (in an arbitrary unit of lenght) of solutions C. Solid (dashed) curve on the right figure corresponds to the positive (negative) r.

Solution C. Finally, let us consider some slightly more sophisticated sources:

$$F^2 = 2\Big[\mathrm{sech}(x^4+r) - \mathrm{sech}(x^4-r)\Big]^2$$
$$-\mathrm{sech}(x^4+r)\tanh(x^4+r) + \mathrm{sech}(x^4-r)\tanh(x^4-r),$$
$$Q^2 = \Big[\mathrm{sech}(x^4+r) - \mathrm{sech}(x^4-r)\Big]^2, \tag{21}$$

where r is a free parameter which determines the distance between two peaks of the source functions F^2 and Q^2 (see Fig. 3). We find that the conformal factor in this case is given by

$$a^2 = \exp\Big\{4\arctan\Big[\tanh(\frac{x^4+r}{2})\Big] - 4\arctan\Big[\tanh(\frac{x^4-r}{2})\Big]\Big\}. \tag{22}$$

The asymptotic behaviours of the conformal factor and the source functions in this case are similar to the previous ones, i.e. at infinity we have the flat Minkowskian space. For positive r, in the region $|x^4| < r$ the conformal factor is a decreasing function as we go away from $x^4 = 0$ point. Therefore, like the Refs. [3] one can expect that gravity is localized at $x^4 = 0$, while now one can place the matter on the "branes" corresponding to the peaks (minima) of F^2 (see, Fig. 3). This situation is similar to multi-brane models of Refs. [5], so that one can also speculate on the solution to the hierarchy problem. For negative r, the conformal factor around $x^4 = 0$ behaves as in [2].

Clearly, plenty of other solutions with different source functions F and Q could be considered as well. Here we have considered relatively simple source functions with shapes which mimic δ-function type sources (thin branes). Obviously, in order to make a final conclusion about the phenomenological implications and the validity of such types of models, a more detailed analysis similar to those [14] done for the previously proposed models [2,3] has to be performed.

We conclude with the following comments. It is remarkable that, while we have concentrated here on five-dimensional case, our approach can be straightforwardly extended to any number of extra dimensions. This is not the case for the proposals [2,3] which are valid for five dimensions and an extension to higher dimensions is possible only within the framework of intersecting branes [6], or within the sting-like objects in the case of two extra dimensions [7]. The next comment concerns the source functions. Needless to say, that the source functions considered above are chosen *ad hoc*, rather than obtained from an underlying dynamics. In its turn it is an interesting and rather non-trivial task to obtain the desired distribution of matter in higher dimensions from some more general theory that ensures the correct behaviour of gravity in four dimensions. Here we can only hope that such a theory can be constructed.

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